Calc III, August 27

§12.2 Vectors

linear shifts of vectors are equivalent, "same vector"
, ie same "direction & magnitude"
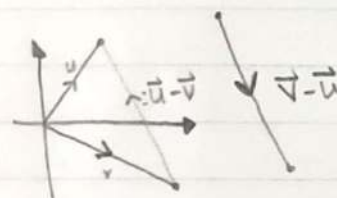
$|\vec{v}|$ = magnitude of $\vec{v}$
easily found by d formula

Direction of Vectors?

Vector Operations:

1. taking magnitude $\vec{v} \mapsto \mathbb{R} \geq 0$
2. adding vectors $\vec{v_6} \mapsto \vec{v}$ (tip to tail)
3. subtracting vectors $\vec{v_5} \mapsto \vec{v}$ (tip to tip)
4. negating vector $\vec{v} \mapsto \hat{v}$
5. scalar multiplication $s \cdot \vec{v} \mapsto \vec{v}$

$\vec{u} - \vec{v}$ $\vec{v} - \vec{u}$

Components of Vectors

every vector has unique representative line segment with tail at origin
↳ given that vector from origin, head coords. determine the vector ie $\langle x, y \rangle$

vector $s \nearrow^t$ has components $t - s = \langle x, y \rangle$

Algebraic Operations (in 3-space)

1) Magnitude

$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$
$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} \quad \subset \text{ distance formula}$$

2) Addition

$$\vec{u} = \langle u_1, u_2, u_3 \rangle \quad \vec{v} = \langle v_1, v_2, v_3 \rangle$$
$$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$$

3) Subtraction
~ (also component-wise)
(same as addition)~

4) Negation
$$\vec{u} = \langle u_1, u_2, u_3 \rangle$$
$$-\vec{u} = \langle -u_1, -u_2, -u_3 \rangle$$

5) Scalar Multiplication
$$c \in \mathbb{R} \quad \vec{u} = \langle u_1, u_2, u_3 \rangle$$
$$c\vec{u} = \langle cu_1, cu_2, cu_3 \rangle$$

Properties of Vector Operations

Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ & $a, b \in \mathbb{R}$

1) $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$ (associativity)
2) $\vec{u}+\vec{v} = \vec{v}+\vec{u}$ (commutativity)
3) $\vec{0}+\vec{v} = \vec{v}$ (identity)
4) $\vec{u}+(-\vec{u}) = 0$ (negatives exist)
5) $a(b\vec{u}) = ab(\vec{u})$
6) $a(\vec{u}) + b(\vec{u}) = (a+b)\vec{u}$
7) $a(\vec{u})+a(\vec{v}) = a(\vec{u}+\vec{v})$
8) $1(\vec{u}) = \vec{u}$ and $0\vec{u} = \vec{0}$

Direction
Property of magnitude $c \in \mathbb{R}$ $\vec{u} \in \mathbb{R}^n$
$$|c\vec{u}| = |c| \cdot |\vec{u}|$$

Def: direction of $\vec{u}$ = associated unit vector (v with length 1)
$$= \frac{1}{|\vec{u}|} \cdot \vec{u} \quad \text{when } \vec{u} \neq 0$$

Claim $\quad \dfrac{1}{|\vec{u}|}\vec{u} \quad$ is unit vector

$$= \left|\dfrac{1}{|\vec{u}|}\vec{u}\right| = \dfrac{1}{|\vec{u}|}|\vec{u}| = 1$$

Component Vectors

$$\left.\begin{array}{l} \vec{i} = \langle 1, 0, 0 \rangle \\ \vec{j} = \langle 0, 1, 0 \rangle \\ \vec{k} = \langle 0, 0, 1 \rangle \end{array}\right\} \text{standard bass for } \mathbb{R}^3$$